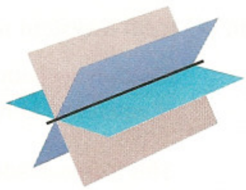
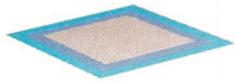
is equation (3). Thus removing equation (3) from the system does not affect the solution of the system.* In writing an answer to this problem, we simply state that "the equations are dependent."

Try Exercise 21.

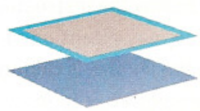
Recall that when dependent equations appeared in Section 3.1, the solution sets were always infinite in size and were written in set-builder notation. There, all systems of dependent equations were *consistent*. This is not always the case for systems of three or more equations. The following figures illustrate some possibilities geometrically.



The planes intersect along a common line. The equations are dependent and the system is consistent. There is an infinite number of solutions.



The planes coincide. The equations are dependent and the system is consistent. There is an infinite number of solutions.



Two planes coincide. The third plane is parallel. The equations are dependent and the system is inconsistent. There is no solution.

3.4 Exercise Set

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Concept Reinforcement *Classify each of the following statements as either true or false.*

1. $3x + 5y + 4z = 7$ is a linear equation in three variables. True

2. It is not difficult to solve a system of three equations in three unknowns by graphing. False

3. Every system of three equations in three unknowns has at least one solution. False

4. If, when we are solving a system of three equations, a false equation results from adding a multiple of one equation to another, the system is inconsistent. True

5. If, when we are solving a system of three equations, an identity results from adding a multiple of one equation to another, the equations are dependent. True

6. Whenever a system of three equations contains dependent equations, there is an infinite number of solutions. False

7. Determine whether $(2, -1, -2)$ is a solution of the system

$$\begin{aligned} x + y - 2z &= 5, \\ 2x - y - z &= 7, \\ -x - 2y - 3z &= 6. \end{aligned}$$

Yes

8. Determine whether $(-1, -3, 2)$ is a solution of the system

$$\begin{aligned} x - y + z &= 4, \\ x - 2y - z &= 3, \\ 3x + 2y - z &= 1. \end{aligned}$$

No

Solve each system. If a system's equations are dependent or if there is no solution, state this.

9. $$\begin{aligned} x - y - z &= 0, \\ 2x - 3y + 2z &= 7, \\ -x + 2y + z &= 1 \end{aligned}$$

$(3, 1, 2)$

10. $$\begin{aligned} x + y - z &= 0, \\ 2x - y + z &= 3, \\ -x + 5y - 3z &= 2 \end{aligned}$$

$(1, 3, 4)$

*A set of equations is dependent if at least one equation can be expressed as a sum of multiples of other equations in that set.

11. $\begin{aligned} x - y - z &= 1,\\ 2x + y + 2z &= 4,\\ x + y + 3z &= 5 \end{aligned}$ $(1, -2, 2)$

12. $\begin{aligned} x + y - 3z &= 4,\\ 2x + 3y + z &= 6,\\ 2x - y + z &= -14 \end{aligned}$ $(-4, 5, -1)$

13. $\begin{aligned} 3x + 4y - 3z &= 4,\\ 5x - y + 2z &= 3,\\ x + 2y - z &= -2 \end{aligned}$ $(2, -5, -6)$

14. $\begin{aligned} 2x - 3y + z &= 5,\\ x + 3y + 8z &= 22,\\ 3x - y + 2z &= 12 \end{aligned}$ $(3, 1, 2)$

15. $\begin{aligned} x + y + z &= 0,\\ 2x + 3y + 2z &= -3,\\ -x - 2y - z &= 1 \end{aligned}$ No solution

16. $\begin{aligned} 3a - 2b + 7c &= 13,\\ a + 8b - 6c &= -47,\\ 7a - 9b - 9c &= -3 \end{aligned}$ $(-3, -4, 2)$

17. $\begin{aligned} 2x - 3y - z &= -9,\\ 2x + 5y + z &= 1,\\ x - y + z &= 3 \end{aligned}$ $(-2, 0, 5)$

18. $\begin{aligned} 4x + y + z &= 17,\\ x - 3y + 2z &= -8,\\ 5x - 2y + 3z &= 5 \end{aligned}$ No solution

Aha! **19.** $\begin{aligned} a + b + c &= 5,\\ 2a + 3b - c &= 2,\\ 2a + 3b - 2c &= 4 \end{aligned}$ $(21, -14, -2)$

20. $\begin{aligned} u - v + 6w &= 8,\\ 3u - v + 6w &= 14,\\ -u - 2v - 3w &= 7 \end{aligned}$ $(3, -5, 0)$

21. $\begin{aligned} -2x + 8y + 2z &= 4,\\ x + 6y + 3z &= 4,\\ 3x - 2y + z &= 0 \end{aligned}$ The equations are dependent.

22. $\begin{aligned} x - y + z &= 4,\\ 5x + 2y - 3z &= 2,\\ 4x + 3y - 4z &= -2 \end{aligned}$ The equations are dependent.

23. $\begin{aligned} 2u - 4v - w &= 8,\\ 3u + 2v + w &= 6,\\ 5u - 2v + 3w &= 2 \end{aligned}$ $(3, \frac{1}{2}, -4)$

24. $\begin{aligned} 4a + b + c &= 3,\\ 2a - b + c &= 6,\\ 2a + 2b - c &= -9 \end{aligned}$ $(-\frac{1}{2}, -1, 6)$

25. $\begin{aligned} r + \tfrac{3}{2}s + 6t &= 2,\\ 2r - 3s + 3t &= 0.5,\\ r + s + t &= 1 \end{aligned}$ $(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$

26. $\begin{aligned} 5x + 3y + \tfrac{1}{2}z &= \tfrac{7}{2},\\ 0.5x - 0.9y - 0.2z &= 0.3,\\ 3x - 2.4y + 0.4z &= -1 \end{aligned}$ $(\frac{3}{5}, \frac{2}{3}, -3)$

27. $\begin{aligned} 4a + 9b &= 8,\\ 8a + 6c &= -1,\\ 6b + 6c &= -1 \end{aligned}$ $(\frac{1}{2}, \frac{2}{3}, -\frac{5}{6})$

28. $\begin{aligned} 3u + 2w &= 11,\\ v - 7w &= 4,\\ u - 6v &= 1 \end{aligned}$ $(4, \frac{1}{2}, -\frac{1}{2})$

29. $\begin{aligned} x + y + z &= 57,\\ -2x + y &= 3,\\ x - z &= 6 \end{aligned}$ $(15, 33, 9)$

30. $\begin{aligned} x + y + z &= 105,\\ 10y - z &= 11,\\ 2x - 3y &= 7 \end{aligned}$ $(17, 9, 79)$

31. $\begin{aligned} a - 3c &= 6,\\ b + 2c &= 2,\\ 7a - 3b - 5c &= 14 \end{aligned}$ $(3, 4, -1)$

32. $\begin{aligned} 2a - 3b &= 2,\\ 7a + 4c &= \tfrac{3}{4},\\ 2c - 3b &= 1 \end{aligned}$ $(\frac{1}{4}, -\frac{1}{2}, -\frac{1}{4})$

Aha! **33.** $\begin{aligned} x + y + z &= 83,\\ y &= 2x + 3,\\ z &= 40 + x \end{aligned}$ $(10, 23, 50)$

34. $\begin{aligned} l + m &= 7,\\ 3m + 2n &= 9,\\ 4l + n &= 5 \end{aligned}$ $(2, 5, -3)$

35. $\begin{aligned} x + z &= 0,\\ x + y + 2z &= 3,\\ y + z &= 2 \end{aligned}$ No solution

36. $\begin{aligned} x + y &= 0,\\ x + z &= 1,\\ 2x + y + z &= 2 \end{aligned}$ No solution

37. $\begin{aligned} x + y + z &= 1,\\ -x + 2y + z &= 2,\\ 2x - y &= -1 \end{aligned}$ The equations are dependent.

38. $\begin{aligned} y + z &= 1,\\ x + y + z &= 1,\\ x + 2y + 2z &= 2 \end{aligned}$ The equations are dependent.

TW **39.** Rondel always begins solving systems of three equations in three variables by using the first two equations to eliminate x. Is this a good approach? Why or why not?

TW **40.** Describe a method for writing an inconsistent system of three equations in three variables.

SKILL REVIEW

To prepare for Section 3.5, review translating sentences to equations (Section 1.7).

Translate each sentence to an equation. [1.7]

41. One number is half another. Let x and y represent the numbers; $x = \frac{1}{2}y$

42. The difference of two numbers is twice the first number. Let x and y represent the numbers; $x - y = 2x$

43. The sum of three consecutive numbers is 100. Let x represent the first number; $x + (x + 1) + (x + 2) = 100$

44. The sum of three numbers is 100. Let x, y, and z represent the numbers; $x + y + z = 100$

45. The product of two numbers is five times a third number. Let x, y, and z represent the numbers; $xy = 5z$

46. The product of two numbers is twice their sum. Let x and y represent the numbers; $xy = 2(x + y)$

SYNTHESIS

TW **47.** Is it possible for a system of three linear equations to have exactly two ordered triples in its solution set? Why or why not?

TW **48.** Kadi and Ahmed both correctly solve the system

$$\begin{aligned} x + 2y - z &= 1,\\ -x - 2y + z &= 3,\\ 2x + 4y - 2z &= 2. \end{aligned}$$

Kadi states "the equations are dependent" while Ahmed states "there is no solution." How did each person reach the conclusion?

Solve.

49. $\begin{aligned} \frac{x+2}{3} - \frac{y+4}{2} + \frac{z+1}{6} &= 0,\\ \frac{x-4}{3} + \frac{y+1}{4} + \frac{z-2}{2} &= -1,\\ \frac{x+1}{2} + \frac{y}{2} + \frac{z-1}{4} &= \frac{3}{4} \end{aligned}$ $(1, -1, 2)$

50. $\begin{aligned} w + x + y + z &= 2,\\ w + 2x + 2y + 4z &= 1,\\ w - x + y + z &= 6,\\ w - 3x - y + z &= 2 \end{aligned}$ $(1, -2, 4, -1)$

51. $\begin{aligned} w + x - y + z &= 0,\\ w - 2x - 2y - z &= -5,\\ w - 3x - y + z &= 4,\\ 2w - x - y + 3z &= 7 \end{aligned}$ $(-3, -1, 0, 4)$